DIAGNOSTIC COMPETENCES OF MATHEMATICS TEACHERS – PROCESSES AND RESOURCES

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Reviewing the research on teachers' diagnostic competences shows that most findings focus on the correspondence between teachers' diagnostic judgments and students' actual achievement, while cognitive processes and cognitive resources of teachers in diagnostic situations have been examined much less. We intend to extend this state of research from a domain-specific point of view by empirically identifying and theoretically describing processes and resources of mathematics teachers while judging tasks and students' solutions. In an interview study with expert teachers and mathematics educators (n=6) it was possible to deduce typical steps in a diagnostic process to identify resources (i.e. aspects of teacher knowledge) they relied on.

INTRODUCTION

In mathematics teaching we find many different diagnostic situations, which can be characterized according to their position in the learning process and their respective objectives (e.g. Ingenkamp & Lissmann 2008; Wiliam 2007):

- *Initial assessment* aims at gaining information about the students' conditions for future learning (e.g. previous knowledge of students).
- *Formative assessment* is needed for supporting individuals or for adapting instructional choices during the learning process.
- *Summative assessment* is needed for assessing learning results and can be used for certification or placement of students.

(Depending on the authors the terms 'assessment' and 'diagnosis' are considered either synonymous or contrasting in certain aspects. In this paper we assume no difference). Diagnostic situations can also be differentiated by the level of formality: In addition to formal diagnostic tests, there are also informal diagnostic situations which influence instruction. In mathematics teaching such diagnostic situations are often linked to the activity of working with tasks, e.g. (i) Teachers analyse and select tasks with respect to their potential diagnostic value and (ii) teachers evaluate students' solutions to a task.

Current and recent research focuses on the precision of teachers' diagnostic judgements (dubbed the 'veridicality-paradigm') (cf. Hoge & Colardaci 1989, Südkamp, Kaiser & Möller 2012), while many questions regarding the cognitive processes of teachers during the assessment process and the domain specificity of diagnostic competence remain unsettled (Schrader, 2011). In a similar way that Ball, Thames & Phelps (2008) investigated mathematical knowledge for teaching by analysing teachers activities, we intend to create some insights into teachers'

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diagnostic competencies by analysing their cognitive processes and their use of resources during assessment.

THEORETICAL FRAMEWORK

Concepts of diagnostic competence

Diagnostic competence most often is defined as the ability of a person to judge people appropriately (Schrader, 2011) and measured by numerical indicators for the precision of such diagnostic judgements. Three aspects of precision are frequently studied, each of them being related to specific diagnostic activities and situations (Spinath 2005, Lorenz & Artelt 2009, Schrader & Helmke 1987): (1) The judgement of a *level* of an attribute of a student or a task relates to the situation of selecting tasks with an appropriate content or level of difficulty. One can ask if teachers underrate or overrate such attributes. (2) Judging the *variance* of some attributes within a group of students is necessary for deciding about individualisation strategies. Finally (3) correctly estimating the *rank* (a) of the difficulty of tasks or (b) the abilities of students can tell something on the use of content knowledge for selecting tasks or the knowledge on the relative strengths and weaknesses of the class. It seems obvious that the numerical precision of such judgments can only be regarded as an indicator for diagnostic competence at work. Within this approach knowledge about the structure and the influencing factors of diagnostic competence (pertaining to the task, the student, the context or the teacher) is based on studying the reasons for judgment biases (Südkamp, Kaiser & Möller 2012).

Still there are many open questions left, such as in which way teachers generate diagnostic judgements in the pedagogical context. There is a lack of understanding of cognitive processes of teachers guiding their judgement. Also the domain-specificity or even topic-specificity of diagnostic competence and how diagnostic competence is composed would be of interest. By correlating the above-mentioned indicators Spinath (2005) showed that diagnostic competence should not be considered as general ability but rather as construct that consists of several sub-competences. Still we do not possess any fairly coherent theoretical model of diagnostic competence and empirical evidence for it (Schrader 2011; Anders et al. 2010).

For mathematics education it is a fruitful task to contribute to a better understanding of the processes and the knowledge connected to diagnostic situations with respect to the domain of mathematics. This can be seen as embedded within the broader challenge of constructing a theory of teacher knowledge in mathematics. For example, within the framework of Ball et al. (2008) competences needed for diagnostic activities can be located in several areas: *Common content knowledge* (CCK) is needed to evaluate the correctness of a student's solution for instance, *specialized content knowledge* (SCK) is used for example to vary the degree of difficulty of tasks and *knowledge of content and students* (KCS) helps to understand students (mis-)conceptions and approaches.

Approaches for modelling diagnostic processes

Understanding cognitive processes in diagnostic situations can also be seen as a question within the field of research on expertise. Here one can find several models for cognitive processes in diagnostic situations which can be used as a framework for further research: Croskerry (2009) proposes a model for diagnostic reasoning in the medical context by integrating previous efforts of promoting diagnostic competence of physicians: (1) the intuitive approach leaning on experience and gestalt effects and (2) the analytic approach using knowledge and systematic information gathering. To describe diagnostic judgements she proposes a dual process model (in the sense of Kahneman, 2003) where patterns are processed by an unconscious system and by rational processes of a conscious system which interact in specific ways (practice, override and calibration processes) to reach a diagnostic judgement. The fact that Croskerry (2009) calls this a “universal” model already indicates that this can be considered a broad framework which leaves many space for specification (such as by modelling the conscious system by critical thinking, training, logical competence etc.). Nickerson (1999) proposes a model to describe the process of rating other people’s knowledge. First a model of own knowledge is used as an anchor to describe the knowledge of others (default model). In several steps this model is refined by including information on the particularity of one’s own position, on the random other and on more and more information on specific others. This way the process of gaining insight in other people’s knowledge can be seen as an alternation of anchoring and adjustment (Tversky & Kahnemann 1974). In this model Nickerson can explain frequent tendencies of overestimating knowledge of others. Nickersons model appears to be very general and especially refers to factual knowledge. It should be transferred into pedagogical context with caution. Morris et al. (2009) on the other hand construct a model very specific to a diagnostic situation in mathematics teaching. They show that “unpacking” the sub-goals of a task can be considered an important facet of diagnostic competence with regard to the planning and evaluation of learning processes. The ability to decompose mathematical content within a task can be useful in diagnostic situations to locate students’ mistakes. However is doubtful if the “unpacking competence” is enough to master diagnostic situations which require identifying deficient conceptions of students, since misconceptions (such as the “division makes smaller” error in calculating with fractions) cannot be deduced by analysing correct solution processes.

These examples of very different scope show, that there are indeed different frameworks available for modelling cognitive processes and knowledge resources of teachers during diagnostic activities. To substantiate these models it seems desirable to have a concrete picture of cognitive processes of mathematics teachers. It is our goal not to test these general models but to create knowledge on processes in the concrete domain of mathematics teaching that can connect to the more general models and inspire further research in this area.

RESEARCH QUESTIONS

In our study we focus on informal diagnostic situations and teachers working with mathematical tasks in a diagnostic way, such as when judging tasks or evaluating students’ solutions. These diagnostic situations often occur when tasks have to be selected and embedded in existing material or when the teacher has to react towards students mistakes spontaneously. For an in-depth investigation of teachers’ diagnostic competence in these situations we assume a double focus on processes and on resources during the formation of diagnostic judgments and pursue the following research questions: (1) What kind of *processes* can be identified in teachers´ diagnostic judgements? (2) What kind of *knowledge* do teachers rely on during these processes? By these questions we intend to create a deeper understanding of diagnostic processes but also to further clarify possible components of diagnostic competence of mathematics educators. A long-term objective connected with our research is to derive consequences for teacher education and professional development.

DESIGN OF THE STUDY

As a method to gain information on cognitive processes and knowledge of teachers we decided to capture their reasoning by means of two phased think-aloud interviews (Ericsson & Simon 1993). In the first phase we initiated diagnostic processes by first presenting two tasks and afterwards three students’ solutions to each task and asking the participants to evaluate each of them. In the second phase teachers had to reflect their own diagnostic process by describing the process and additionally by giving reasons for their judgement. By this combination of parallel and retrospective think we expected to capture a large part of the relevant processes.

As participants we chose three experienced mathematics teachers and three scholars in mathematics education. The latter had experiences as mathematics teachers *and* as teacher educators (for at least three years in each of their professional phases) and so we could draw on practical experience and reflected theoretical knowledge likewise. The aim of selecting this sample was to find a maximum variety of different processes. Think-aloud-protocols of the diagnostic processes and the reflections of their own processes supplied the data for the analysis in the present study which amounted to 12 evaluations of tasks and 36 evaluations of students’ solutions. For the interviews we chose the tasks from the topic “fractions”, because of the broad systematic knowledge about students’ conceptions, errors and misconceptions in this field. The tasks and the interview guideline were developed and optimized in a pilot study. The students´ solutions were selected so that they represented typical mistakes and frequent misconceptions. Figure 1 shows the tasks and solutions we used.

In the first phase the participants had to analyse tasks. They were asked: “Which challenges do you see? Which difficulties do you expect?” Then the participants were given the three students’ solutions to each task and had to evaluate them by answering the question: “Which conclusions do you draw?”

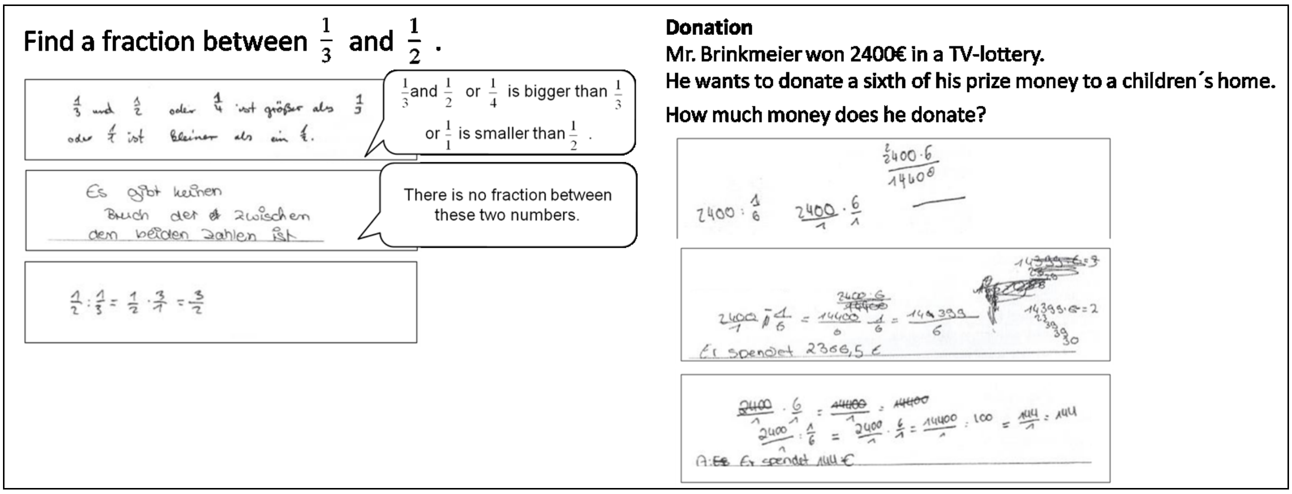

Figure 1: Tasks and students´ solutions (from: Wartha, 2007).

Interpretative content analysis techniques were used to analyse the data (Ericsson & Simon 1993, Mayring 1983) with the objectives were to reconstruct types of diagnostic processes and to generate a theoretical overarching structure. In the analysis we first focused on assessment processes (see research question 1). In the next step we analysed the same data with a focus on the kinds of knowledge underlying these processes (see research question 2).

RESULTS

We present some exemplary results of the two interpretative cycles described above. Focussing on diagnostic processes (see research question (1)) resulted in more than 15 Processes, of which we present three important ones. Table 1 shows the name of the process (code), a description of the code and excerpt of an interview to illustrate the category.

Code	Description	Representative teacher statement
Standard solution	Design a solution for a given task.	“[...] you can solve it by division.”
Identify deficits	Discover and name an incorrect approach.	“1/4 is bigger than 1/3. This is typical. When the numbers are in the denominator. With bigger and smaller.”
Identify strengths	Discover and name skills.	“[...] this is great. He writes down the number 2400 as fraction.”

Table 1: Excerpt of identified assessment processes.

The category “standard solution” refers to the process of designing a solution on your own or mentioning a common solution approach by its name. “Identify deficits” refers to recognizing an incorrect approach in a student solution. Finally to “identify strengths” means to see students´ competences in their solutions. When analysing the same data with a focus on the kinds of knowledge underlying these processes (see

research question (2)) we found among others the following knowledge categories: In the first example in Table 1 (see above) the interviewee refers to the mathematical correctness and therefore draws on his mathematical knowledge. In the second example the interviewee explicitly refers to a typical mistake and therefore uses a component of pedagogical content knowledge that refers to systematic knowledge (gained in educational research). In the third example one can see a reference to a concept of a fraction as a rational number – although it remains unclear whether this should be assigned to explicit knowledge on students' development of number concepts or merely to the recognition of a mathematical fact.

When analysing the results of the coding process (which could only be indicated by few examples here) it is possible to develop a "bigger picture": Some of the processes are essential; they show up frequently and can be interpreted as steps in an assessment process. In every step different qualities of individual processes were observable.

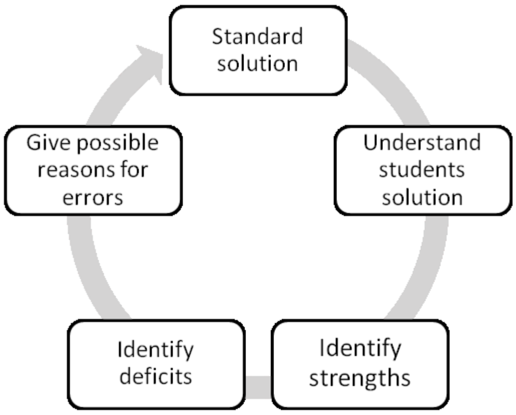

Figure 2: Idealized five-step model for the diagnostic process.

Figure 2 shows an idealized model of steps during informal assessment: The initial point often is a standard solution or an approach. Then own solutions are compared with students' solutions. Thereby strengths and deficits of the solution can be identified. The last step is to find a (hypothetical) reason (or several reasons) for errors – if they occur. As a very common strategy across all steps we observed that interviewees spontaneously decompose tasks or solutions and to analyse them step by step – just as Morris et al (2009) advise the participants in their study.

The cognitive resources the interviewees rely on when moving through the diagnostic process as described above can be characterized as different types of knowledge: We could identify content knowledge (CK) and pedagogical content knowledge (PCK). For example, the participants of our study used knowledge on *mental models* of mathematical concepts, on *typical errors* and on typical *misconceptions*, Furthermore *mathematical correctness* was evaluated and *student strategies* were identified.

CONCLUSIONS AND DISCUSSION

The main objective of our study was a deeper understanding of diagnostic processes. Although such processes followed quite individual patterns, they could be categorized as different types of "steps in the diagnostic process". Furthermore it became evident

that the participants showed different degrees of flexibility, for example in the number of possible approaches to solve a task they mentioned (often combined with more than one representation). In future analyses this flexibility may serve as an indicator for the quality of the diagnostic process and/or the diagnostic competence of the teacher. To clarify this connection remains an open question for further study.

Another objective was to identify types of knowledge, which teachers use while forming their diagnostic judgement – this amounts to delineate different components of diagnostic competence. A provisional interpretation of our results with regard to aspects of diagnostic competence is that three different aspects can be identified: (1) *Knowledge*: the use of content knowledge (CK) as well as pedagogical content knowledge (PCK) was observable. (2) *Abilities*: we observed the ability to decompose mathematical tasks but also to analyse tasks and solutions step by step and the ability to take the students' perspective. (3) *Attitudes*: we state that a kind of readiness for assessment is necessary e.g. for taking students perspective – although this aspect did not emerge directly by our systematic analysis but is inferred rather generally from our experience during the interviews.

We regard our results as modest extensions to theoretical frameworks which only partially focus on diagnostic competence, First our results regarding aspects of knowledge can be integrated into the theoretical framework of Ball et al. (2008) but still need further foundation, e.g. by efforts to quantitatively measure the aspects described here. Second, the decomposing of tasks and students solutions found within our study can be considered close to the research by Morris et al. (2009). However, while Morris et al. refer only to teachers "unpacking" mathematical concepts one should also consider the process of teachers identifying misconceptions that cannot be deduced by starting from correct mathematical concepts.

Finally our research also uncovered certain differences in diagnostic processes of teachers with different levels of experience. For example, in our analyses some striking differences showed up, which also should be investigated further: Experts (with a scholarly background) seem to use a variety of different approaches to analyse a task. They appear to be more focused on strengths in their assessment than teachers. Experts draw more explicitly on subject-based knowledge (PCK). Because of the sample size of our study these differences can be seen as tendencies only. They should be regarded as hypotheses which need a more rigorous treatment and may be tested in a different design.

References

Anders, Y., Kunter, M., Brunner, M., Krauss, S., & Baumert, J. (2010). Diagnostische Fähigkeiten von Mathematiklehrkräften und ihre Auswirkungen auf die Leistungen ihrer Schülerinnen und Schüler. *Psychologie in Erziehung und Unterricht*, *57*, 175-193.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, *59*, 389-407.

Croskerry, P. (2009). A universal model of diagnostic reasoning. *Academic Medicine*, *84*(8), 1022-1028.

Ericsson, K. A., & Simon, H. A. (1993). *Protocol analysis: Verbal reports as data* (Rev. ed.). Cambridge, Mass: MIT Press.

Hoge, R. D., & Coladarci, T. (1989). Teacher-based judgments of academic achievement: A review of literature. *Review of Educational Research*, *59*, 297-313.

Ingenkamp, K., & Lissmann, U. (2008). *Lehrbuch der pädagogischen Diagnostik.* Weinheim, Basel: Beltz.

Kahneman D. (2003). A perspective on judgment and choice. *American Psychologist*, *58*(9), 697-720.

Lorenz, C., & Artelt, C. (2009). Fachspezifität und Stabilität diagnostischer Kompetenz von Grundschullehrkräften in den Fächern Deutsch und Mathematik. *Zeitschrift für Pädagogische Psychologie, 23*(3-4), 211-222.

Mayring, P. (1983). *Qualitative Inhaltsanalyse: Grundlagen u. Techniken*. Weinheim; Basel: Beltz.

Morris, A. K., Hiebert, J., & Spitzer, S. M. (2009). Mathematical knowledge for teaching in planning and evaluating instruction: What can preservice teachers learn? *Journal for Research in Mathematics Education*, *40*(5), 491-529.

Nickerson, R. S. (1999). How we know-and sometimes misjudge-what others know: Imputing one's own knowledge to others. *Psychological Bulletin*, *125*(6), 737-759.

Schrader, F.-W. (2011). Lehrer als Diagnostiker. In E. Terhart, H. Bennewitz, & M. Rothland (Eds.), *Handbuch der Forschung zum Lehrerberuf* (pp. 683-698). Münster: Waxmann.

Schrader, F.-W., & Helmke, A. (1987). Diagnostische Kompetenz von Lehrern: Komponenten und Wirkungen. *Empirische Pädagogik*, *1*, 27-52.

Spinath, B. (2005). Akkuratheit der Einschätzung von Schülermerkmalen durch Lehrer und das Konstrukt der diagnostischen Kompetenz. *Zeitschrift für Pädagogische Psychologie*, *19*, 85-95.

Südkamp, A., Kaiser, J., & Möller, J. (2012). Accuracy of teachers' judgments of students' academic achievement: A meta-analysis. *Journal of Educational Psychology*, *104*(3), 743-762.

Tversky, A., & Kahnemann, D. (1974). *Judgement under uncertainty: heuristics and biases. Science*, *185*, 1124-1131.

Wartha, S. (2007). *Längsschnittliche Untersuchungen zur Entwicklung des Bruchzahlbegriffs.* Hildesheim: Franzbecker.

Wiliam, D. (2007). Keeping learning on track. In F. K. J. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1053-1098). Charlotte, NC: Information Age.